ISSN 2229-5518

Stability of Parametrically Driven Dark Lattice Solitons in Nanoscale Systems

O.P. Swami*, Vijendra Kumar, A.K. Nagar

Abstract— In this paper, we consider a parametrically driven nanoscale device modelled by discrete nonlinear Schrodinger equation. To determine the stability of fundamental dark solitons, analytical and numerical calculations are performed. We show that a parametric driving can change the stability of dark solitons. Stability windows of fundamental dark solitons are presented and stability approximations are derived using perturbation theory, with numerical results.

Index Terms— Discrete nonlinear Schrodinger equation, Nanoelectromechanical system, Nanoscale device, Parametrically driven, Perturbation theory, Solitons, Stability.

1 Introduction

We consider a nanoelectromechanical system (NEMS) governed by a parametrically driven discrete nonlinear Schrodinger (PDNLS) equation

$$i\dot{\Psi}_n = -C\Delta_2\Psi_n + \left|\Psi_n\right|^2\Psi_n - \Omega\Psi_n - \alpha\overline{\Psi}_n \quad (1)$$

where $\Psi_n \equiv \Psi_n(t)$ is a real-valued wave function and 'n' is the lattice site index. The overdot and the overline denote the time derivative and the complex conjugation, respectively. C is the coupling constant between two adjacent sites, $\Delta_2\Psi_n = \Psi_{n+1} + \Psi_{n-1} - 2\Psi_n$ is the one-dimensional (1D) discrete Laplacian, α is the parametric driving coefficient with frequency Ω. Parametrically driven electromechanical resonators have been discussed in Ref. [1, 2]. Discrete bright soliton type systems have been discussed before e.g. in Ref. [3-7]. In undriven case Eq. (1) reduces to the standard discrete nonlinear Schrodinger (DNLS) equation, which appears in many applications [8]. The same equation also applies to the study of discrete modulational instability in parametrically driven optical lattice [9]. The long bosonic Josephsen junction and BEC trapped Optical lattices are also studied using same equation [10, 11, 12].

In this paper we examine the condition for stability of fundamental onsite dark soliton in defocusing PDNLSE.

For small C, the perturbation theory is used, followed by numerical computations in MATLAB.

2 Analytical Setup And Perturbative Results

Stationary solution of system (1) in the form of $\Psi_n = X_n$, where X_n is a time independent and real valued wave function, X_n satisfies the stationary equation

$$-C\Delta_2 X_n + X_n^3 - \Omega X_n - \alpha X_n = 0 \quad (2)$$

To examine the stability of X_n, we introduce the linearization ansatz

$$\Psi_n = X_n + \delta Y_n$$

where $\delta \ll 1$, and substitute this in to Eq.(1), it yield the following linearization equation at $O(\delta)$:

$$iY_n = -C\Delta_2 Y_n + 2\left|X_n\right|^2 Y_n + X_n^2\overline{Y_n} - \Omega Y_n - \alpha Y_n \quad (3)$$

writing $Y_n = A_n + iB_n$, and linearizing in δ, we find

$$\begin{pmatrix} \dot{A}_n \\ \dot{B}_n \end{pmatrix} = N \begin{pmatrix} A_n \\ B_n \end{pmatrix} \quad (4)$$

where

$$N = \begin{pmatrix} 0 & M_+ \\ -M_- & 0 \end{pmatrix} \quad (5)$$

and

$$M_+(C) = -C\Delta_2 + (X_n^2 - \Omega + \alpha)$$

$$M_+(C) = -C\Delta_2 + (3X_n^2 - \Omega - \alpha)$$

Let the eigenvalues of N be denoted by id, which implies that X_n is stable if Im(d)=0. Since Eq.(5) is linear, we can eliminate one of 'eigenvectors', for instance B_n, then we obtain the following eigenvalue problem

$$M_+(C)M_-(C) = d^2 A_n = \Lambda A_n \quad (6)$$

3 Analytical Calculation

In the uncoupled limit $C = 0$, we denote the exact solutions of (2) by $X_n = X_n^{(0)}$, in which each $X_n^{(0)}$ must take one of three values given by $0, \pm\sqrt{\Omega + \alpha}$.

Following Ref.[1], using a perturbative expansion, the dark soliton solutions are obtained as

$$X_n = \begin{cases} -\sqrt{\Omega+\alpha} + \frac{1}{2}C/\sqrt{\Omega+\alpha}, & n = -1 \\ 0, & n = 0 \\ \sqrt{\Omega+\alpha} - \frac{1}{2}C/\sqrt{\Omega+\alpha}, & n = 1 \end{cases} \quad (7)$$

and its eigenvalues for small C are given by

- *O. P. Swami is currently a research scholar in material science at Department of Physics, Govt. Dungar College, Bikaner, Rajasthan 334001, India, PH-07737131150. E-mail: omg1789@gmail.com*
- *Vijendra Kumar is currently a research scholar in material science at Department of Physics, Govt. Dungar College, Bikaner, Rajasthan 334001, India, PH-08447491889. E-mail: vijendrasaini2009@gmail.com*
- *A. K. Nagar, Ph.D. in material science is professer at Department of Physics, Govt. Dungar College, Bikaner, Rajasthan 334001, India, PH-09829348569. E-mail: ajaya.nagar@gmail.com*

$$\Lambda = \Omega^2 - \alpha^2 - 4\Omega C + O(C^2) \quad (8)$$

The instability of onsite discrete dark soliton is due to the collision of the smallest eigenvalue (8) with an eigenvalue bifurcating from lower and upper edge of continuous spectrum, for small and large α, respectively. Equating these quantities, we find the critical value of α as a function of the coupling constant C i.e

$$\alpha_{cr}^1 = -0.4\Omega - 1.6C + 0.2\sqrt{9\Omega^2 - 28\Omega C - 16C^2} \quad (9)$$

$$\alpha_{cr}^2 = \sqrt{\Omega^2 - 4\Omega C} \quad (10)$$

Both α_{cr}^1 and α_{cr}^2, give approximate boundaries of the instability region in (C,α) plane.

4 Comparison With Numerical Calculation

Using Newton-Raphson method, we have numerically solved the static equation (2), and analyzed the stability of the numerical solution by solving the eigenvalue problem (4). We consider $\Omega = 0.9$ in the model.

Figure 1 provides a full description of the dynamics of the parametrically driven DNLS model regarding the intervels of stability/instability of the model. Analytical predictionbility range as obtained by the conditions of collision of the phase mode eigenfrequency with the continuous spectrum from Eqs. (9)-(10).

Figure 2,3 illustrate the typical instability scenario for different values of parametric drives α and coupling constant C. Where the panel-a present the structure of just before the collision (stable), whereas the penal-b represents just after the collision (unstable).

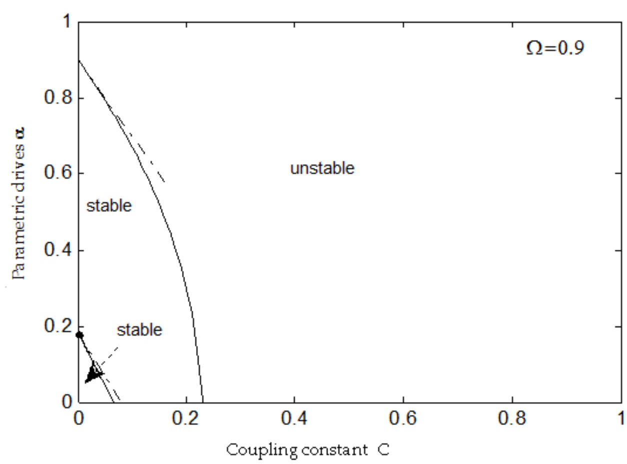
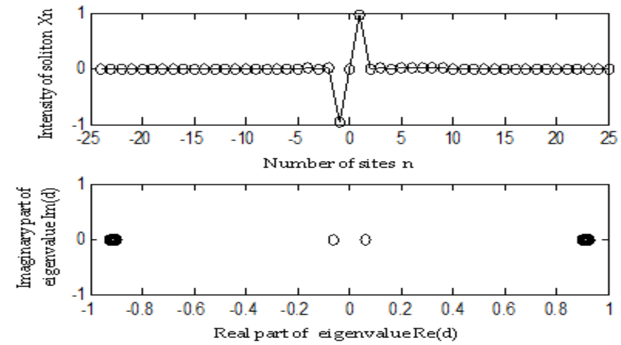
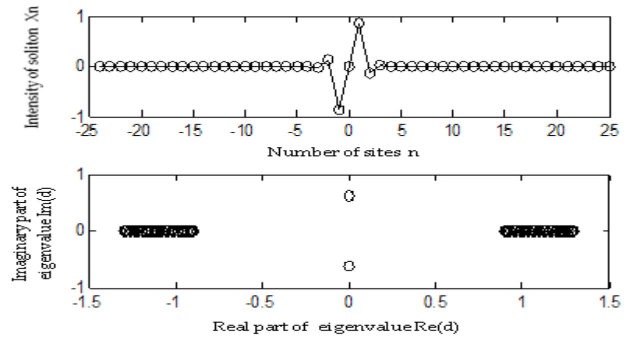


Fig. 1. The stability-instability region in the two parameter space $\alpha - C$. The solid lower and upper lines are the analytical approximations of Eqs. (9) and (10). Lower and upper dash-dotted lines are their respective numerical approximations.

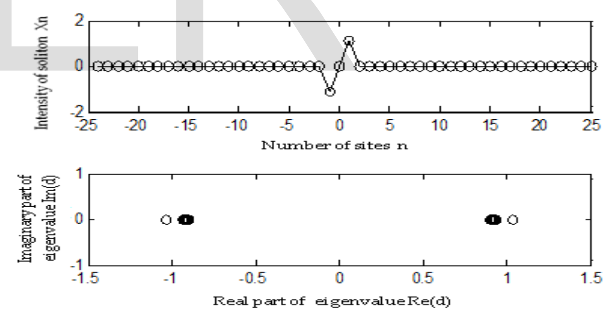


(Panel-a)

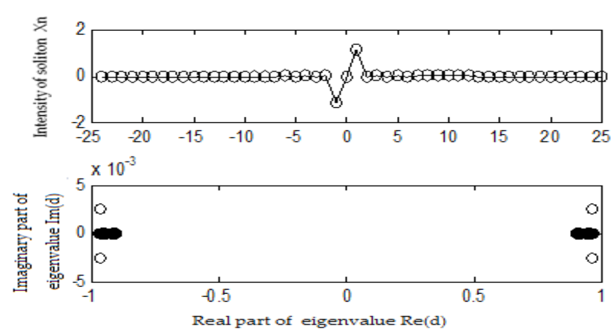


(Panel-b)

Fig. 2. The eigenvalue structure of onsite dark soliton for $\alpha = 0.03$ and $C = 0.008$ (panel-a), as well as $C = 0.03$ (panel-b).



(Panel-a)



(Panel-b)

Fig. 2. The eigenvalue structure of onsite dark soliton for $\alpha = 0.4$ and $C = 0.01$ (panel-a), as well as $C = 0.028$ (panel-b).

ISSN 2229-5518

5 CONCLUSIONS

In this paper, we have considered a parametricallydriven Nano Electro Mechanical System, which is modeled by DNLS. The stability of fundamental onsite dark solitons are determined using perturbative analysis, which is followed by Numerical Computations in MATLAB. We have shown that the presence of parametric driving can change the stability of governed model. It destabilizes the onsite dark soliton. We have considered the frequency of parametric drive as $\Omega = 0.9$, which is smaller than that in Ref[1]. The result is a downward shift in $C-\alpha$ graph (Fig.1), which is expected from analysis.

REFERENCES

[1] M. Syafwan, H. Susanto, and S. M. Cox, *Phys. Rev.* E81, 026207, 2010.

[2] M. Syafwan, "The existence and the stability of solitons in discrete nonlinear Schrodinger equation", Ph.D. Thesis, Department of Physics, Nottingham University, 2012.

[3] O. P. Swami, V. Kumar, and A. K. Nagar, *Int. J. Mod. Phy.* 22, pp. 570-575, 2013.

[4] H. Susanto, Q. E. Hoq, and P. G. Kevrekidis, *Phys. Rev.* E4, 067601-4, 2006.

[5] D. Hennig and G. Tsironis, *Phys. Rev.* 307, pp. 333-432, 1999.

[6] G. L. Alfimov, V. A. Brazhnyi, and V. V. Konotop, *Phys. Rev.* D194, pp. 127-150, 2004

[7] D. E. Pelinopsky, P. G. Kevrekidis, and D. J. Frantzeskakis, *Physica.* D212, pp. 1-19, 2005.

[8] P. G. Kevrekidis, *Discrete Nonlinear Schrodinger Equation: Mathematical Analysis Numerical Computations and Physical Perspectives*, New York: Springer, pp. 145,192-194, 2009.

[9] O. P. Swami, A. Sharma, and A. K. Nagar, "Discrete modulational instability in parametrically driven optical lattices", *AIP Conference Proceedings 1536, American Institute of Physics,* Melville, NY, pp. 757-758, 2013.

[10] V. Kumar, O. P. Swami, A. K. Nagar, "Existance and stability of dark solitons in Bose-Einstien condensate in parabolic trapped optical lattices", *International Journal of Scientific & Engineering Research,* Vol. 5, Issue 3, March-2014, pp. 329-331, ISSN 2229-5518.

[11] V. M. KAurov, and A. B. Kuklov, *Phys. Rev.* A71, 011601, 2005.

[12] V. M. KAurov, and A. B. Kuklov, *Phys. Rev.* A73, 013627, 2006.